TRIBHUVAN UNIVERSITY

Institute of Science and Technology

2067

Bachelor Level/ First Year/ First Semester/ Science — Full Marks: 80

Computer Science and Information Technology (MTH 104) — Pass Marks: 32

(Calculus and Analytical Geometry) — Time: 3 hours.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all questions.

Group A (10x2=20)

1. Define a relation and a function from a set into another set. Give suitable example.

2. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converses by using integral test.

3. Investigate the convergence of the series $\sum_{n=0}^{\infty} \frac{2^n+5}{3^x}$.

4. Find the foci, vertices, center of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

5. Find the equation for the plane through (-3, 0, 7) perpendicular to $\vec{n} = 5\vec{\imath} + 2\vec{\jmath} - \vec{k}$.

6. Define cylindrical coordinates (r, v, z). Find an equation for the circular cylinder $4x^2 + 4y^2 = 9$ in cylindrical coordinates.

7. Calculate $\int\int_R f(x,y)d4$ for $f(x, y) = 1 - 6x^2y$, $R : 0 \leq x \leq 2$, $-1 \leq y \leq 1$.

8. Define Jacobian determinant for $x = g(u, v, w)$, $y = h(u, v, w)$, $z = k(u, v, w)$.

9. What do you mean by local extreme points of $f(x, y)$? Illustrate the concept by graphs.

10. Define partial differential equations of the first index with suitable examples.

Group B (5x4=20)

11. State the mean value theorem for a differentiable function and verify it for the function $f(x) = \sqrt{1 - x^2}$ on the interval [-1, 1].

12. Find the Taylor series and Taylor polynomials generated by the function $f(x) = \cos x$ at $x = 0$.

13. Find the length of cardioid $r = 1 - \cos\theta$.

14. Define the partial derivative of $f(x, y)$ at a point (x_0, y_0) with respect to all variables. Find the derivative of $f(x, y) = xe^y + \cos(x, y)$ at the point (2, 0) in the direction of $A = 3i - 4j$.

15. Find a general solution of the differential equation

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z.$$

Group C (5x8=40)

16. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x - axis and the line $y = x - 2$.

OR

Investigate the convergence of the integrals

(a) $\int_1^0 \frac{1}{1-x}\, dx$ (b) $\int_0^3 \frac{dx}{x-1^{2/3}}$

17. Calculate the curvature and torsion for the helix $r(t) = (a \cos t)i + (a \sin t)j + b\, t\, k$, $a, b \geq 0$, $a^2 + b^2 \neq 0$.

18. Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

19. Find the absolute maximum and minimum values of $f(x, y) = 2 + 2x + 2y - x^2 - y^2$ on the triangular plate in the first quadrant bounded by lines $x = 0$, $y = 0$ and $x + y = 9$.

OR

Find the points on the curve $xy^2 = 54$ nearest to the origin. How are the Lagrange multipliers defined?

20. Derive D' Alembert's solution satisfying the initials conditions of the one-dimensional wave equation.